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New Family of Exotic Θ-Baryons

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From the interpretation of the Θ^+ baryon resonance as an excitation of the "skyrmion liquid" with SU(3) flavor symmetry $\overline{10}$ we deduce a new series of baryons, Θ_1^{++}, Θ_1^+ and Θ_1^0, situated at the top of the 27-plet of SU(3) flavor, with hypercharge $Y = 2$, isospin $T = 1$ and spin $J = \frac{3}{2}$. The mass of Θ_1 is estimated 55 MeV/c^2 higher then the mass of Θ^+ and its width at 80 MeV. We also discuss the other baryons from the 27-plet.

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Recently an exotic and narrow baryon resonance, Θ^+, which cannot be formed by three quarks was observed in three independent experiments [1, 2, 3]. Masses of 1540 ± 10 MeV/c^2 [1], 1539 ± 2 MeV/c^2 [2] and 1543 ± 5 MeV/c^2 [3] were reported, in excellent agreement with the theoretical prediction $M^{\rm th}_{\Theta^+} = 1530$ MeV/c^2 [4]. In these experiments the resonance width was estimated at $<$ 25 MeV [1], $<$ 9 MeV [2] and $<$ 22 MeV [3] comparable with the theoretical prediction $\Gamma^{\rm th}_{\Theta^+} < 15$ MeV [4]. The theoretical predictions for the Θ^+-baryon were done in the framework of the extended Skyrme model for the SU(3) flavor multiplet $\mu = (0, 3)$ with the dimension $N_\mu = \overline{10}$ (anti-decuplet) [20]. The hypercharge of the observed Θ^+, $Y = 2$, follows from strangeness conservation in electromagnetic and strong interactions, the isospin cannot be determined from experiments [1, 2, 3]. If Θ^+ is associated with the top of the anti-decuplet its other quantum numbers must be $T = 0$ and $J^P = \frac{1}{2}^+$.

Contrary to the picture, where Θ^+ is considered as an excitation of a "skyrmion liquid" with appropriate SU(3) flavor symmetry, this resonance can also be interpreted as Fock-state component $uudd\bar{s}$ [11]. In this pure multi-quark picture Θ^+ has isospin, spin and parity different from that predicted by the Skyrme model, e.g., Θ^+ can be an isotensor resonance with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ or $\frac{5}{2}^-$.

Besides exotic baryons in the anti-decuplet the Skyrme model predicts other SU(3) flavor multiplets with exotic baryons. A first estimate for the nearest partners of Θ^+ shows that these must be exotic states in the $\mu = (2, 2)$ representation (dimension $N_\mu = 27$) with quantum numbers $Y = 2$, $T = 1$ and $J^P = \frac{3}{2}^+$. Depending on the fit of the known baryon spectra its mass was estimated between 100 and 150 MeV/c^2 larger than the mass of the Θ^+ [12]. To clarify the situation it is important to make a detailed study of the predictions of the Skyrme model for the baryons from the anti-decuplet together with baryons from higher multiplets and to give, if possible, new predictions, which can support or reject the soliton picture for the nature of the Θ-baryon.

In the present short paper we calculate the mass spectrum of the baryons from the 27-plets with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ and compare them with experiment. We find that the $J^P = \frac{1}{2}^+$ baryons are systematically 500 MeV/c^2 heavier than the $J^P = \frac{3}{2}^+$ baryons. We show that besides two additional exotic resonances (which we call Γ and Π) with hypercharge-isospin (Y, T) = (0,2) and (-1,$\frac{3}{2}$), respectively, there are new families of Θ-baryons, Θ_1 and Θ_2, with (Y, T) =(2,1). The lightest of them should be only 55 MeV/c^2 heavier than the Θ^+-baryon. Θ_1 has a typical hadronic width Γ_{Θ_1} ∼80 MeV.

Starting from a hedgehog ansatz and assuming rigid rotation in SU(3) space [13, 14] one obtains the following Hamiltonian for the baryon representation $\mu = (p, q)$ of the SU(3) flavor group

$$\begin{aligned} H =& M_0 + \frac{1}{6I_2}[p^2 + q^2 + pq + 3(p + q)]+ \\ &+ \left(\frac{1}{2I_1} - \frac{1}{2I_2}\right) \hat{\vec{J}}^2 - \frac{(N_c B)^2}{24I_2} + \Delta\hat{H}, \end{aligned} \tag{1}$$

where $\hat{\vec{J}}$ is the spin operator, M_0 is the energy of a static soliton solution, I_1 and I_2 are the two moments of inertia, $N_c = 3$ is the number of colors and $B = 1$ is the baryon number. All quantities M_0, I_1 and I_2 are functionals of the soliton profile. The Hamiltonian $\Delta\hat{H}$ is responsible for the splitting within SU(3) multiplets [14]

$$\Delta\hat{H}(R) = \alpha D^{(8)}_{88}(R) + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{A=1}^{3} D^{(8)}_{8A}(R)\hat{J}_A. \tag{2}$$

Here $D^{(8)}_{mn}(R) = \frac{1}{2}{\rm Tr}\left(R^\dagger\lambda_m R\lambda_n\right)$ are Wigner rotation matrices for the adjoint SU(3) representation. The constants α, β and γ are related to the current quark masses, m_u, m_d, m_s, the nucleon sigma term and four soliton moments of inertia [4, 14].

Due to the Wess-Zumino term the quantization rule selects only such soliton spins J, which coincide with one of the allowed isospins T for hypercharge $Y = 1$ in the

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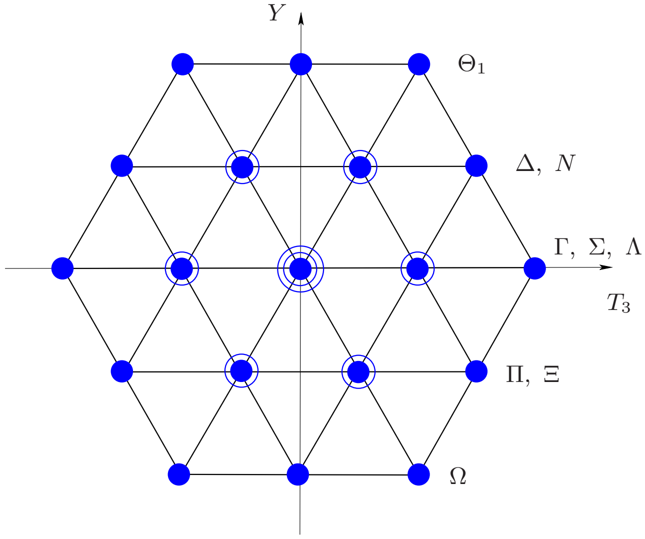

FIG. 1: Structure of the 27-plet of baryons in the T_3Y diagram.

given SU(3) flavor multiplet [13, 14]

$$J = T \qquad \text{for} \qquad Y = 1. \tag{3}$$

So the lightest irreducible SU(3) representations which can be (very roughly) associated with 3-quark and 4-quark–antiquark systems and with allowed spins J are

$$\begin{array}{lll} \text{octet} & \mu=(1,1) & J=1/2 \\ \text{decuplet} & \mu=(3,0) & J=3/2 \\ \text{anti-decuplet} & \mu=(0,3) & J=1/2 \\ \text{27-plet} & \mu=(2,2) & J=1/2 \text{ or } 3/2 \\ \text{35-plet} & \mu=(4,1) & J=3/2 \text{ or } 5/2 \end{array} \tag{4}$$

The wave functions for baryons with hypercharge Y, isospin T, isospin 3-projection T_3, spin J and its z-projection J_3 depend on 8 parameters (similar to Euler angles in SU(2)) of the SU(3) manifold

$$\langle R|\mu YTT_3JJ_3\rangle = \sqrt{N_\mu}(-1)^{J_3-\frac{1}{2}} D^{\mu}_{YTT_3;1J-J_3}(R), \tag{5}$$

where N_μ is the dimension of the representation μ. From the Hamiltonian (1) we get the mass spectrum

$$\begin{aligned} M =& M_0 + \frac{1}{6I_2}[p^2+q^2+pq+3(p+q)]+ \\ &+\left(\frac{1}{2I_1}-\frac{1}{2I_2}\right)J(J+1) - \frac{3}{8I_2} + \Delta M, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Delta M &= \langle \Delta\hat{H}\rangle = \\ &= \int dR \langle \mu YTT_3JJ_3|R\rangle \Delta\hat{H}(R)\langle R|\mu YTT_3JJ_3\rangle. \end{aligned} \tag{7}$$

The rotational energy is given by the second and third terms in (6). In general it increases very strongly from the octet representation in (4) to the 35 representation. But there is one exception. From numerical results it follows that $I_1 > I_2$. This means that in (6) the term proportional to $J(J+1)$ become more negative for higher angular momenta. So moving from the $J=\frac{1}{2}$ anti-decuplet to the $J=\frac{3}{2}$ 27-plet the increase of the rotational energy of the second term in (6) can be compensated by the increase of the negative contribution of the third term. Estimates with typical parameters for the moments of inertia I_1 and I_2 show that the rotation energy increases by ≈ 100 MeV only, which, in principal, is of the order of the splitting within the SU(3) multiplet! The structure of the 27-plet of baryons in the T_3Y diagram is displayed in Fig. 1. The states Θ_1, Γ_{27} and Π_{27} are exotic and due to their Y and/or T values cannot be reduced to three quark systems.

The SU(3) Clebsch-Gordan coefficients needed for calculations of the mass splitting in the 27-plet were taken from [15]. These splittings are given in Table I together with the results of Ref. [4] for the anti-decuplet.

In our numerical calculations we use the following parameters from Ref. [4]

$$\begin{aligned} &I_2 = (500 \text{ MeV})^{-1}, \\ &\alpha = -218 \text{ MeV},\ \beta = -156 \text{ MeV},\ \gamma = -107 \text{ MeV}. \end{aligned} \tag{8}$$

The first moment of inertia I_1 was estimated from the experimental masses of the baryons from the $\frac{1}{2}^+$ octet

TABLE I: Mass splitting in anti-decuplet with $J=\frac{1}{2}$ and in 27-plets with $J=\frac{1}{2}$ and $\frac{3}{2}$.

anti-decuplet[a]	T	Y	ΔM
Θ^+	0	2	$(1/4)\alpha+2\beta-(1/8)\gamma$
$N_{\overline{10}}$	1/2	1	$(1/8)\alpha+\beta-(1/16)\gamma$
$\Sigma_{\overline{10}}$	1	0	0
$\Xi_{\overline{10}}$	3/2	-1	$-(1/8)\alpha-\beta+(1/16)\gamma$
27-plet $J=3/2$			
Θ_1	1	2	$(1/7)\alpha+2\beta-(5/14)\gamma$
Δ_{27}	3/2	1	$(13/112)\alpha+\beta-(65/224)\gamma$
N_{27}	1/2	1	$(1/28)\alpha+\beta-(5/56)\gamma$
Γ_{27}	2	0	$(5/56)\alpha-(25/112)\gamma$
Σ_{27}	1	0	$-(1/56)\alpha+(5/112)\gamma$
Λ_{27}	0	0	$-(1/14)\alpha+(5/28)\gamma$
Π_{27}	3/2	-1	$-(1/14)\alpha-\beta+(5/28)\gamma$
Ξ_{27}	1/2	-1	$-(17/112)\alpha-\beta+(85/224)\gamma$
Ω_{27}	1	-2	$-(13/56)\alpha-2\beta+(65/112)\gamma$
27-plet $J=1/2$			
Θ_1	1	2	$(17/56)\alpha+2\beta-(1/112)\gamma$
Δ_{27}	3/2	1	$(1/28)\alpha+\beta-(5/112)\gamma$
N_{27}	1/2	1	$(137/560)\alpha+\beta+(71/1120)\gamma$
Γ_{27}	2	0	$-(13/56)\alpha-(19/112)\gamma$
Σ_{27}	1	0	$(13/280)\alpha+(19/560)\gamma$
Λ_{27}	0	0	$(13/70)\alpha+(19/140)\gamma$
Π_{27}	3/2	-1	$-(17/112)\alpha-\beta+(1/224)\gamma$
Ξ_{27}	1/2	-1	$-(23/280)\alpha-\beta+(63/560)\gamma$
Ω_{27}	1	-2	$(1/14)\alpha-2\beta+(5/28)\gamma$

[a] From Ref. 4.

and the Σ^* from the $\frac{3}{2}^+$ decuplet

$$I_1 = \frac{2}{3(m_{\Sigma^*} - m_\Sigma - m_N + m_\Lambda)}. \quad (9)$$

M_0 we get from the mass of the nucleon. The estimated masses are given in Table II. Concerning the $J^P = \frac{1}{2}^+$ 27-plet we would like to mention that its states are approximately 500 MeV higher than the states of the $J^P = \frac{3}{2}^+$ 27-plet.

Neglecting transitions to the 35-plet (which is around 1 GeV higher than the $J^P = \frac{3}{2}^+$ 27-plet) the states

$$\Theta_1,\ N_{27},\ \Gamma_{27},\ \Lambda_{27},\ \Pi_{27},\ \text{and}\ \Omega_{27} \quad \text{with}\ J = \frac{3}{2} \quad (10)$$

exist as pure members of the 27-plet. The states Δ_{27}, Σ_{27} and Ξ_{27} should be mixed with the corresponding decuplet states. Therefore, their wave functions read

$$\begin{aligned} |\Delta\rangle &= |\Delta_{10}\rangle + C_\Delta |\Delta_{27}\rangle, \\ |\Sigma\rangle &= |\Sigma_{10}\rangle + C_\Sigma |\Sigma_{27}\rangle, \\ |\Omega\rangle &= |\Omega_{10}\rangle + C_\Omega |\Omega_{27}\rangle, \end{aligned} \quad (11)$$

where the admixture coefficients are given by

$$C_B = \frac{\langle B_{10}|\Delta\hat{H}|B_{27}\rangle}{M_{27} - M_{10}}, \quad M_{27} - M_{10} = \frac{1}{I_2}. \quad (12)$$

The transition amplitudes read

$$\begin{aligned} \langle \Delta_{10}|\Delta\hat{H}|\Delta_{27}\rangle &= \frac{\sqrt{30}}{16}\left(\alpha + \frac{5}{6}\gamma\right), \\ \langle \Sigma_{10}|\Delta\hat{H}|\Sigma_{27}\rangle &= \frac{1}{4}\left(\alpha + \frac{5}{6}\gamma\right), \\ \langle \Xi_{10}|\Delta\hat{H}|\Xi_{27}\rangle &= \frac{\sqrt{6}}{16}\left(\alpha + \frac{5}{6}\gamma\right). \end{aligned} \quad (13)$$

Using the parameters (8) and (9) one gets the following admixtures between the $J = \frac{3}{2}$ 27-plet and the decuplet

$$C_\Delta = -0.210,\ C_\Sigma = -0.154,\ C_\Xi = -0.094. \quad (14)$$

This mixture for $J = \frac{3}{2}$ baryons is huge, larger than the mixture between $J = \frac{1}{2}$ baryons in the octet and the anti-decuplet which was shown to be universal and equal to $C_{8-10} = 0.084$ [4]. Such a strong mixture shows that one cannot ignore the $\overline{q}qqqq$ component in strong and electromagnetic transitions between nucleons and deltas.

Because of the non-vanishing transition amplitudes (13) we get second order corrections to the mass spectrum of the decuplet

$$\begin{aligned} \Delta m_\Delta^{(2)} &= -\frac{15}{128} m_2, \\ \Delta m_{\Sigma^*}^{(2)} &= -\frac{1}{16} m_2, \\ \Delta m_{\Xi^*}^{(2)} &= -\frac{3}{128} m_2, \\ \Delta m_\Omega &= 0, \quad m_2 = I_1 \left(\alpha + \frac{5}{6}\beta\right)^2. \end{aligned} \quad (15)$$

TABLE II: Mass spectrum in anti-decuplet with $J^P = \frac{1}{2}^+$ and in 27-plet with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$.

particle	J	Theory (MeV/c^2)	Experiment
Θ^+	1/2	1530 [e]	1540 ± 10 [a]; 1539 ± 1 [b]; 1542 ± 5 [c]
$N_{\overline{10}}$	1/2	1710 (input)[e]	$N(1710)P_{11}$ [d]
$\Sigma_{\overline{10}}$	1/2	1890[e]	$\Sigma(1660)P_{11}$ [d]
$\Xi_{\overline{10}}$	1/2	2070[e]	$\Xi(1690)(?)$ or $\Xi1950(?)$[d]
Θ_1	3/2	1595	—
Δ_{27}	3/2	1750	$\Delta(1600)P_{33}$ [d]
N_{27}	3/2	1746	$N(1720)P_{13}$ [d]
Γ_{27}	3/2	1904	—
Σ_{27}	3/2	1899	—
Λ_{27}	3/2	1896	$\Lambda(1890)P_{03}$ [d]
Π_{27}	3/2	2052	—
Ξ_{27}	3/2	2048	$\Xi(1690)(?)$ or $\Xi1950(?)$[d]
Ω_{27}	3/2	2200	$\Omega(2250)(?)$[d]
Θ_2	1/2	2028	—
Δ_{27}	1/2	2246	$\Delta(1910)P_{31}$ [d]
N_{27}	1/2	2189	—
Γ_{27}	1/2	2474	—
Σ_{27}	1/2	2411	—
Λ_{27}	1/2	2350	—
Π_{27}	1/2	2594	—
Ξ_{27}	1/2	2569	—
Ω_{27}	1/2	2682	—

[a]From Ref. 1.
[b]From Ref. 2.
[c]From Ref. 3.
[d]From Ref. 16.
[e]From Ref. 4.

These corrections lead to violations of the equidistance in the spectrum and to the following sum rules

$$\begin{aligned} & m_{\Sigma^*} + m_{\Xi^*} - m_\Delta - m_\Omega = \\ & = 2(2m_{\Xi^*} - m_{\Sigma^*} - m_\Omega) = \\ & = 2(2m_{\Sigma^*} - m_\Delta - m_{\Xi^*}). \end{aligned} \quad (16)$$

For an equidistant spectrum every line in (16) would give zero. But experimentally these quantities do not vanish. Inserting the experimental masses in these three lines results in 15 MeV/c^2, 16 MeV/c^2 and 13 MeV/c^2. This is in good agreement with the sum rules (16) derived from the second order corrections independent from the model parameters (8) and (9). But inserting the values (8) and (9) in the second order corrections (15) leads to deviations (16) from the equidistance two times smaller than in the experimental spectrum.

The second order corrections decrease the masses of the nucleon and Σ in the octet by 5 MeV/c^2.

Using the baryon-meson coupling [17]

$$-i\frac{3G_0}{2m_B}\sum_{A=1}^{3} D_{mA}^{(8)} p_A, \quad (17)$$

we have estimated the width of the Θ_1-resonance in the non-relativistic limit. In (17) $\vec{p}$ is the meson momentum in the resonance frame and the coupling constant $G_0 \approx 19$. The new family of Θ-baryons, contrary to the Θ^+, has normal hadronic width, $\Gamma \approx 80$ MeV.

In conclusion, we predict that there exists a new isotriplet of Θ-baryons, Θ_1^{++}, Θ_1^{+} and Θ_1^{0}, with hypercharge $Y = 2$ and $J^P = \frac{3}{2}^+$. Its mass and width, 1595 MeV/c^2 and 80 MeV, respectively, are predicted from the SU(3) Skyrme model using the same parameters as Diakonov et al. [4] employed for the exotic Θ^+ baryon which was recently observed experimentally [1, 2, 3]. The triplet of Θ_1 baryons is a member of the 27-dimensional representation of the SU(3) flavor group. We identify other non-exotic members of this representation (Δ_{27}, N_{27}, Λ_{27} and, possibly, Ξ_{27}) with observed resonances, but do not see a structure, which can be related to the Σ_{27} resonance. Further we predict two additional exotic resonances, Γ_{27} and Π_{27}. It is shown that there exist strong mixtures between the decuplet and the 27-plet for states with quantum numbers of Δ, Σ and Ξ. These mixtures may be responsible for small violations of the equidistance in the decuplet spectra.

When the paper was finished there appeared an article by Jaffe and Wilczek [18] where they propose that the Θ-baryon "lies in a near-ideally mixed $\mathrm{SU}(3)_\mathrm{f}$ $\overline{10}_\mathrm{f} \oplus 8_\mathrm{f}$". The predicted spectrum differs essentially from the prediction of the Skyrme model.

In another article, which appeared at the same time, Θ^+ was discussed from the point of view of QCD sum rules [19]. A series of pentaquark states with isospin 0,1 and 2 with $J^P = \frac{1}{2}^-$ is predicted to lie close to each other near 1550 MeV. In the Skyrme model we have also close resonances, $\Theta(1540)$ and $\Theta_1(1595)$, but with positive parity and spin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, as given in Table I.

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